Large Area Solar Heating System

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Heating with a sun collector costs nothing and contaminates nothing. In this article I propose a solar heating system (SHS) consisting of a large solar collector, suitable for the heating of a house and a heat store, capable to bridge a week of cloudy weather. Both are made from simple materials.

1 Introduction

When we buy oil for the stove we buy heat for about 5 c/kWh. For an electric stove the price goes up to something like 20 c/kWh as a result of the more complicated energy conversions from the power plant up to the house: fuel → heat → electricity → heat. Prices are typical for NL at the time of writing. With the proposed solar heating system (SHS) we need no more to worry about these things. I hope it will become a revolution in heating, at least in the Mediterranean region, because the design is meant for Spain[1] where the radiation over a whole year corresponds to 1770 hours full sun. Full sun is defined as (= 1 kWh /m^2).

The SHS consists of a solar collector and a heat store. Water, with its large heat capacity, serves for transport and for storing the heat. It flows within closed circuits between the collector and the heat store and between the heat store and the radiators in the house. For reasons of hygiene the water *used* in the house (drinking, washing) should come from a separate source and the hot part should pass through the store via a heat exchanger. This can be long tube passing through the store.

The store should be very well insulated in order not to lose much heat during a long period of cloudy weather. The insulation requirements on the collector are less stringent because it can be closed when the sun disappears which stops the heat loss[2].

The starting points in the design are the amount of heat needed, the intensity of the solar radiation, the length of the interval without sun to be bridged and the temperature used in the system relative to the ambient temperture, ΔT. From the first two follows the area of the collector. From the same energy consumption, the length of the interval and ΔT follows how big the store and how thick the layer of insulation should be. As far as the choice of ΔT concerns: Both the ease of heating and the total heat capacity of the store are proportional to ΔT however, also the heat loss. We have chosen a safe value: $\mathbf{\Delta T = 50\,^\circ C}$.

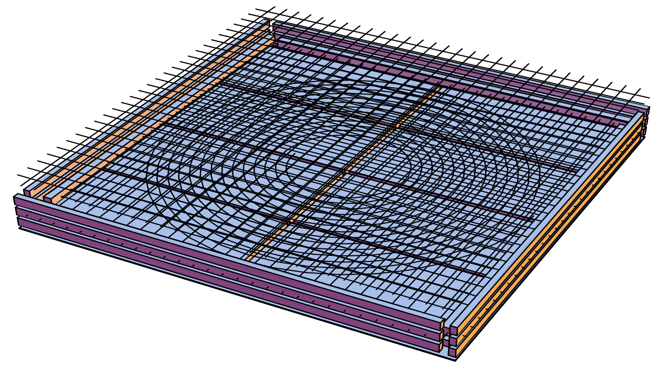

Figure 1: *Test version of the collector with dimensions 2 x 2 m. The spiral represents 250 m tube of the type normally used for irrigation, 16 mm black PVC which absorbs the radiation. Water passes through it to transport the heat. The in- and outlet are not shown. Above the tube are two layers of thin plastic foil, supported by a wiring creating layers of stagnant air. Under the tube and aside is a thick layer of extruded polystyrene foam. About a dozen of these units will be needed for the heating of a house during the winter.*

2 The collector

A common central heating system usually has a power around 20 kW. The Costa del Azahar is less cold and therefore I suppose that half this power and only during a part of the day will suffice. So the starting point is 10 kW

[1] The data used refers to Castellon de la Plana between Tarragona and Valencia (Costa del Azahar)

[2] A disadvantage of a collector that also serves as heat store is that it is difficult to obtain good insulation for the transparent part

during 10 hours a day, or 100 kWh /day = 360 MJ/day [3].

The average solar radiation en January[4] in Castellon on a panel inclined by 60^o is 15.4 MJ/m^2/day (= 4.17 kWh/(m^2 day). Hence even in the ideal case without heat losses we need a large collector area of 24 m^2.

In section 2.2 we will see that the efficiency of a solar collector drops proportionally with ΔT while of course we want a high value to heat the house. ***As a compromise we take a much larger area of 48 m^2*** to relax on the requirement of efficiency. With such a large collector an efficiency of 50 % is enough.

2.1 Thermal insulation of the collector

A difference in temperature between the two sides of a layer of material leads to a heat stream through the material. The thermal current is proportional to the thermal conductivity $U = \lambda/d$ of the material, in which λ is the specific thermal conductivity and d is the thickness of the layer. This is the law of Fourier, similar to Ohms law for electric current. Of course the thermal current is also proportional to de temperature difference and to the area A. So the thermal current i.e. the loss, is

$$Q = \frac{A\,\Delta T}{R_t} \tag{1}$$

$$R_t = \frac{1}{U} = \frac{d}{\lambda} \tag{2}$$

The chosen notation shows better that in order to increase the thermal resistance R_t one should increase d and take a material with a low value of λ. We chose polystyrene foam for this purpose[5].

The upper side of the collector has to be transparent. We will insulate this part with two gaps of 10 cm stationary air. To obtain these gaps we use two layers of thin plastic foil and something that supports it for example chicken wire. Two gaps like this have a thermal conductance of $U \approx 3.5\ W/(m^2K)$. [6]

The lower part is much easier to insulate so we can ask for a loss much less, let's say we accept $0.5\,W/(m^2K)$ in order to arrive at a total conductance (loss) of $4\,W/(m^2K)$ corresponding to a loss of 200 W/m^2 at $\Delta T = 50^o$. From eqs. 1 and 2 follows that ***the required thickness of the polystyrene foam layer is 8cm*** ($\lambda = 0.04,\ \lambda/0.5 = 0.08$).

[3] 1 kWh = 3.6 MJ

[4] In July at 30^o inclination the radiation reaches 23 MJ/m^2/day

[5] Expanded Polystyrene Foam(EPS) $\lambda = 0.033 \cdots 0.042$ price 5.60 euro/m^2 at 4 cm thickness. Source: http://huis-en-tuin.infonu.nl/wonen/17820-isolatiewaarde-k-waarde-u-waarde-lambda-waarde.html

[6] source: www.ekbouwadvies.nl/bouwbesluit

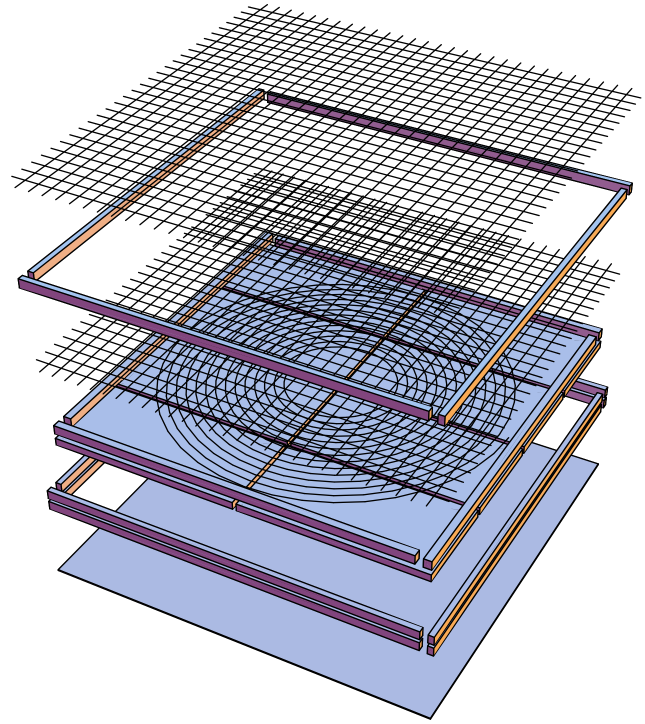

Figure 2: *Exploded view of the 2 x 2 m test version of the proposed collector. From bottom to top: bottom cover, wooden side bars, insulation layer for bottom and sides on which the tube in spiral, plastic foil with chicken wire, more wooden side bars, upper plastic foil with chicken wire*

2.2 Temperature and Efficiency

Note that the larger ΔT the lower the efficiency. In the extreme case that we don't subtract any heat from the collector - efficiency zero - while the radiation is 1000 W/m^2, after some time ΔT will rise to 200 ^{o}C. [7] With our choice of $\Delta T = 50\,^o C$, the loss is 200 W/m^2 hence, at 1000 W irradiation, the efficiency is 80 % . Note that ΔT is always adjusted by means of the water flow.

In winter the sun gives a little more than 4 kWh /m^2/day distributed over about 6 hours. Our test collector, using a ΔT de 50^o, loses during these 6 hours in total 1.2 kWh /m^2 hence we can drain almost 2.8 $kWh\,/m^2$/day or 11.2 kWh/day from the test version of 4 m^2. That's enough to heat 200 liter water 50 ^{o}

[7] May the values for the insulation of the gaps are somewhat optimistic. A reason more to cary out an experiment before construction a full size collector. Anyway *a protection system against overheating is necessary because the isolation foam melts at 120^{o}C*! This protection system can also serve to increase the efficiency when it not only provides cooling water when necessary but also closes the circuit when the collector temperature is below that of the store.

2.3 Heat transport with water

When we make the bottom of the gap in which the spiral tube is situated black this layer will absorb almost all the radiation. However, if we heat the air in this gap and the heat should go from this stagnant air to the tube with water then a much too high temperature gradient is needed to transport the $1000\,W/m^2$ into the tube. The heat conductance over this transition is only 11 $W/(m^2K)$ so we would need an *extra* ΔT of almost $100\,^oC$. This would be disastrous for the efficiency. It is really needed that the sun falls directly on the tube. A solution is to wind a long tube of PVC of 16 mm diameter, normally used for irrigation, in a spiral somewhat distorted towards a rectangle with rounded corners and with minimal space between the turns. This "spiral" is placed on top of the foam insulation layer so that the radiation of the sun reaches it across the upper compartment with stagnant air made by the layers of plastic foil. For a collector unit of 2 m by 2 m, a compromise between ease of handling and efficiency, about 250 m tube is needed.

To transport the 4 kW falling on the collector unit with water (C=4.2 kJ/l/gr) towards the heat store, a flow of only 1.1 liter/minute is enough at a ΔT de $50\,^oC$. Fortunately the heat transport from the wall of the tube to the streaming water is much better ($U = 50\,W/(m^2\,K)$. The internal area of the tube is $6.3\,m^2$ so that the ΔT between the tube and the water is only $13\,^oC$ at a radiation level of 1000 W/m^2. Nevertheless this effect increases the heat loss by 26 %.

May be this extra loss can be compensated for by using the low pressure fall per unit. For 1.1 liter/min in 250 m tube of 16 mm diameter the pressure fall is less than 0.5 Bar. It makes sense in case of a set of many collector units, to put sets of units in series with higher flow, provided that enough pressure is available. This way the units where the water comes in work at a lower ΔT and thus with a higher efficiency.

3 The heat store

3.1 The form

The heat store should be able to store the heat during a long time. The quantity of heat stored is proportional to the volume V, The heat loss is proportional to the outer area A. Hence, the ratio A/V (Area/Volume) should be as small as possible. Let's focus on some common geometrical forms.

3.1.1 A sphere with radius r

$$\left(\frac{A}{V}\right)_{sph} = \frac{4\pi\,r^2}{\frac{4}{3}\pi\,r^3} = \frac{3}{r} \qquad (3)$$

3.1.2 A cube with edge a

$$\left(\frac{A}{V}\right)_{cub} = \frac{6\,a^2}{a^3} = \frac{6}{a} \qquad (4)$$

In a certain sense A/V for a cube with edge a is the same as for a sphere with radius $r = a/2$ because the one fits in the other. It's true that a sphere occupies less space, but the "remaining" space is curved which is in general difficult to use.

3.1.3 A Cylinder with radius r and height h

$$\left(\frac{A}{V}\right)_{cyl} = \frac{2\pi\,r^2 + 2\pi\,r\,h}{\pi\,r^2\,h} = \frac{2}{h} + \frac{2}{r} \qquad (5)$$

With $h = r$ el quotient is $4/r$ y with $h = 2\,r$ it is $3/r$, the same as for a sphere.

We chose a cylinder for its simplicity of construction: The horizontal planes do not require support and the side walls can be fortified with steel cable against the hydrostatic pressure sideways. [8,9].

Fig. 3, calculated with eq. 5, helps to chose de dimensions of a cylinder with a fixed volume. On the horizontal axis is the diameter. The red line, marked "h" gives the height corresponding to a volume of $17\,m^3$ (This volume is calculated in section 3.2) and the blue line gives the ratio A/V. For example at $d = 2.7$ we see that also $h = 2.7$ were A/V has its minimum value 2 or A=2 x $17\,m^2$. We will see further in subsec 3.3 that it is advantageous to lower the height in order to reduce the lateral pressure. Fig. 3 shows that, fortunately, A/V is not very sensitive to a change in h over a factor 2.

3.2 Calculation of the volume required

We calculate the required volume of the heat store to bridge a week without sun. The starting point in section 2 was a heat consumption of 360 MJ per day from which follows an energy drain from the store during a week of 2520 MJ. The specific heat of water is $C_h = 4.2\,MJ/(m^3K)$ thus, with a $\Delta T = 50^o - 13^o$ (see footnote

[8]N.B. For all forms is true that the larger the better; The volume increases and the ratio A/V decreases with size

[9]A truncated pyramid of wet sand might be an alternative solution, attractive because it can be self supporting

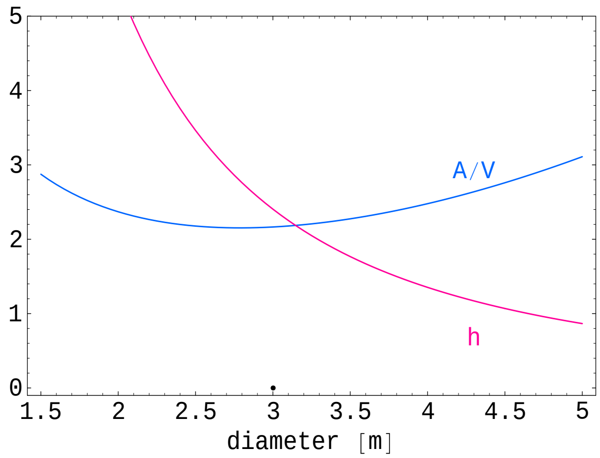

Figure 3: *Compromise between diameter and height for a cylindrical heat store with 17m*^{3} *water. On the horizontal axis is the diameter, the red line gives the corresponding height, the blue line gives the relative area* A/V. *At* $h = d = 2.70\,m$ *the ratio* A/V *is minimal. Note that one can chose a much larger diameter and lower height in order to reduce the hydrostatic pressure without increasing much the ration* A/V

[10]) we need $V = W_0/(C_h\, h\, \Delta T) = 16.2\,\mathrm{m}^3$ of water. In this reasoning we did not take into account any losses but also neglected that in this week, very probable, the sun is still present from time to time. It seems not unreasonable to expect that the two effects cancel.

3.3 Side pressure

When we support the side walls with steel cables we need to know how thick these cables should be. This we derive as follows: The pressure at the bottom of the cylinder is $P = h\,g\,\rho$ with h the height, g the acceleration of gravity and ρ the specific mass of water. According to the law of Pascal this is also the lateral pressure. This pressure decreases linearly with h and thus the average is half that on the bottom. Now cut the cylinder vertically in two equal parts. The force on one half is the average pressure times the area of the cross section $A = 2\,r\,h$. The force is counteracted on two sides hence for one side the force is[11].

$$F_{zijw} = \frac{1}{2} g\,r\,h^2\,\rho \qquad (6)$$
$$\approx 5000\,r\,h^2 \qquad (7)$$

Here we see the argument for a larger d and a much lower h because the side pressure is proportional to r^{-3}. This is seen by substituting $h = V/(\pi\, r^2)$ in eq 6.

F_{zijw} reaches almost 18 000 N for a store of $17\,\mathrm{m}^3$ with $d = 4\,\mathrm{m}$ and $h = 1.35\,\mathrm{m}$. if we divide this force by N wires, placed equidistantly the force is 2 x 18 000 /N on the wire at the bottom and (almost) zero on the top wire. For steel the elastic limit is 250 N/mm^2 that's to say: with 10 equidistant wires the wire at the bottom should have a cross section of at least $36\,000/10/250 = 14.6\,\mathrm{mm}^2$ corresponding to a diameter of 3.8 mm.

3.4 Thickness of the insulation layer

Let's require that without sun, without loading, after a week the store still contains 80% of its original energy. Then

$$W = W_0\, e^{-\alpha\, N_d} = 0.80\, W_0 \qquad (8)$$
$$\rightarrow \alpha = \frac{0.223}{N_d} \qquad (9)$$

in which $N_d = 7$ days and the unit for time is "day". This applies also to the "80 % decay constant" α. A is the area of the proposed heat store with a volume of 17 m^3. ($A/V = 2.5 \rightarrow A = 42.5$).

The loss per day is $\delta W/\Delta T = \alpha W_0$. The thickness d of the insulating layer follows from the equation for the heat flow per day through the layer

$$\alpha\, W_0 = \frac{A\,\Delta T\,K\,s_{pd}}{d} \qquad (10)$$
$$\rightarrow d = \frac{A\,\Delta T\,K\,s_{pd}}{W_0\,\alpha} \qquad (11)$$

in which S_{pd} is seconds per day, K the thermal conductivity (0.04 for polystyrene foam) and d the thickness of the layer. With the above mentioned data *the required thickness is 9 cm.*

4 Possibilities in colder regions

In fig. 4 one can compare the radiation at four european cities: London, Limoges, Barcelone and Sevilla.[12] It is

[10]We have estimated that in house the temperature is 13^o higher than outside. This part of ΔT can not be used to transport heat from the store to the house

[11]The gravitational energy of the store is its mass times the height of the center of mass. We express this energy in V: volume and s: circumference, $E = 2\,g\,\pi\,\rho\,V^2/s^2$. The force is $F = -\delta E/\delta s$, as if the side wall is cut through vertically and held by a wire feeling the pressure. Substitution of V and s gives again equation 6

[12]source: http://re.jrc.ec.europa.eu/pvgis/

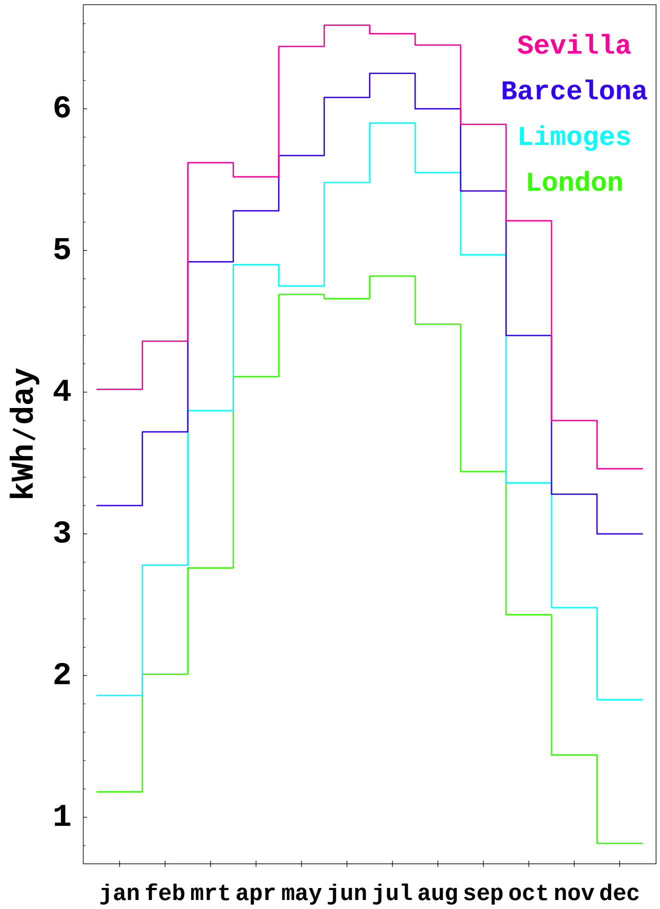

Figure 4: *Equivalent sun hours at optimum inclination at four places in Europe. London, Limoges, Barcelona and Sevilla. Its clear that in London one has to wait until march for a level comparable to that in Barcelona in january.*

clear that in London in winter the proposed system cannot heat the house between november and march. If one still wants to solve this problem he can think of three options

- Double the size of the collector in both dimensions. That brings us in januari qua sunhours at the level of spain.
- Increase the size and isolation of the heat store: Make all dimensions three times as large. This increases the volume by a facor $3^3 = 27$, good for storing heat for half a year, provided that also the "decay constant" α is decreased by a factor 27. This is seen in equation 9. Further $\alpha\, W_0$ in eq. 11 remains unchanged due to the 27 fold increase in W_0. It than follows that d should be taken 9 times larger so we arrive at an isolation layer of *almost a meter thick.*
- Keep the stove on from november to januari.

Increasing the collector area seems the simplest solution but may provoke two problems. One is that in winter in London the periods of bad weather may be longer than one week. Another is that it can only work above a mimimum radiation level, about 200 W/m^2, because that is lost due to the ΔT of 50^o. Increasing the capacity of the heat store certainly will work but is costly and takes much space.

5 Conclusion

For the heating of a house we need a collector size of about $50\, m^2$. We have proposed a test version of $4\, m^2$ to verify if we indeed can drain 11 kWh in a sunny day in winter and with a water temperature 50^o higher than the ambient temperature. When the test version works we need about a dozen of these units or a similar solution with the same exposed area. Next one can start by heating the house directly from the collector. Houses with thick walls already function somewhat like a store and when the sun is away it's not unusual that the temperature drops no faster than five degrees en 24 hours.

Of course, the system gets much more convenient and adjustable with a heat store. We propose a cylindrical store of 4 m in diameter and 1.35 m tall containing 17 m^3 of water. The insulation on all walls has a thickness of 9 cm of a material with $K \leq 0.04$ W/(gr.m). The ratio A/V is 15 % higher than the minimum value (at $h = 2.70$ m but the advantages are: The side wall needs not be very strong, the store can serve as a swimming pool[13] in summer for the children and it looks better in nature.

The proposal above is meant for regions where in januari the radiation from the sun is equivalent to 4 kWh/(m^2/day. In the London region one has to wait until april before this level is reached. Some solutions are given but are quite complicated except may be keeping on the stove from november to march.

Most probable is that the recovery time of the money invested is rather short. Certain is that the favorable effect on the ambient is immediate.

[13] May be a practical solution to make a heat store is to buy a inflatable swimming pool of $17\, m^3$ and construct an insulating box around it